

Chapter 2
Section 2.4

Activity

A college student is hired to deliver new telephone books and collect the old ones for recycling. She is paid \$6 per hour plus \$.30 for each old phone book she collects. Her salary is a function of the number of phone books collected. However, she must use her own car for this job. She figures that her car expenses average \$0.20 per phone book collected plus a fixed cost of \$20 a week for insurance. We can write her expenses as a function of phone books collected as well. Her profit for one week is her salary minus her expenses. So we can also write her profit as a function of phones books collected!

- a) If x is the number of phone books she collected, then what is her salary as a function of x .

$$S(x)=$$

- b) Write her expenses as a function of phone books collected.

$$E(x)=$$

- c) Write her profit as a function of phone books collected.

$$P(x)= \quad - \quad =$$

Basic Operations with Functions

Definition: For two functions f and g , the **sum**, **difference**, **product** and **quotient functions**, are the functions $f + g$, $f - g$, $f \cdot g$, and f/g , respectively, and are defined as follows:

$$\begin{aligned}(f + g)(x) &= f(x) + g(x) \\(f - g)(x) &= f(x) - g(x) \\(f \cdot g)(x) &= f(x) \cdot g(x) \\(f/g)(x) &= f(x)/g(x) \quad \text{provided that } g(x) \neq 0.\end{aligned}$$

Exercise: Let $h = \{(1, 3), (2, 8), (3, 6), (5, 9)\}$ and $f = \{(1, 6), (2, 11), (3, 0), (4, 1)\}$. Let $j(x) = \sqrt{x}$ and $g(x) = 3x - 1$. **Find each function and state its domain.**

- a) $h + f$
- b) $h \cdot f$
- c) h/f
- d) $j + g$
- e) $j \cdot g$
- f) j/g

Composition of Functions

Definition: If f and g are two functions, the **composition** of f and g , written $f \circ g$, is defined by the equation

$$(f \circ g)(x) = f(g(x)),$$

provided that $g(x)$ is in the domain of f . The composition of g and f , written $g \circ f$, is defined by

$$(g \circ f)(x) = g(f(x)),$$

provided that $f(x)$ is in the domain of g .

Exercise: Let $f(x) = \sqrt{x}$, $g(x) = 2x - 1$ and $h(x) = x^2$. **Find each composition and state its domain.**

- a) $f \circ g$
- b) $g \circ f$
- c) $h \circ f$
- d) $h \circ g \circ f$
- e) $f \circ g \circ h$

Question: Is it true or false that $f \circ g = g \circ f$?

Exercise: Let $f(x) = \sqrt{x}$, $g(x) = x - 3$ and $h(x) = 2x$. Write each given function as a composition of appropriate functions chosen from f , g and h .

- a) $F(x) = \sqrt{x - 3}$
- b) $G(x) = x - 6$
- c) $H(x) = 2\sqrt{x} - 3$

Composition with Formulas

Exercise: The radius of a circle is a function of the diameter ($r = d/2$) and the area is a function of the radius ($A = \pi r^2$). Construct a formula that expresses the area as a function of the diameter.

Exercise: A student's salary (in dollars) for collecting x phone books is given by $S(x) = 0.30x + 240$. The amount of withholding (for taxes) is given by $W(x) = 0.20x$, where x is the salary. Express the withholding as a function of the number of phone books collected.